Non-monotonic modal logics of minimal knowledge

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1 Introduction

The interplay of non-monotonic reasoning and reasoning about knowledge and belief has been recognized from the beginning of the research on common sense reasoning. Since then modal epistemic logics have been studied with the aim of characterizing the reasoning of an agent who is capable to perform introspective reasoning by making assumptions on its own knowledge. The first formalizations of non-monotonic reasoning based on the use of a modal operator have been proposed in [10, 12]. The knowledge of an agent is characterized in terms of a fixed-point equation that expresses its introspective capabilities.

A different approach for defining non-monotonic modal logics was taken in [1, 17, 5], where the knowledge of the agent is characterized on a semantic ground, following the idea of selecting those models in which knowledge is minimal.

Recently, there have been a number of attempts to reconcile the fixed-point and semantic characterizations of modal non-monotonic logics. In particular, in [14] the family of non-monotonic modal logic generated by the fixed-point equation of [10] has been given a semantic counterpart. In addition, in [16] a correspondence between the two approaches has been found for a non-monotonic extension of modal logic S4F.

One unsatisfactory aspect of this characterization is that it does not cover the case of the logic S5, whose fixed-point non-monotonic extension collapses into S5 itself, while the semantic approach based on minimization of knowledge leads to the non-monotonic modal logic developed in [1].

The goal of our work is to fill this gap by explaining the reasons of the failure of the proposed fixed-point approaches in the case of logic S5, and by providing a new way to characterize a family of modal non-monotonic logics of minimal knowledge.

The main result we have obtained is the correspondence between a preference semantics which is a genealization of Lifschitz's preference criterion on S5-models shown in [5] and a family of fixed-point logics known as ground non-monotonic logics [18, 2, 15].

In the next section we briefly discuss the logics based on the minimization of knowledge, then we discuss the fixed-point characterizations of non-monotonic modal logics. Finally we present a possible-world semantics for ground non-monotonic logics, a family of fixed-point modal logics whose models are minimal knowledge models.

2 Minimization of Knowledge

In this section we consider the logics based on the intuition that in the models the knowledge attributed to the agent should be minimal. This principle was introduced in [1] and it was enforced by minimizing the set of objective facts known by the agent. In its most general version, this notion of minimal knowledge can be stated as follows (we use $\mathcal{L}$ to denote a fixed propositional language, $\mathcal{L}_K$ to denote its modal extension with the only modality K; the set of formulas $I \subseteq \mathcal{L}_K$ stands for the initial knowledge of the agent and $Th(\mathcal{M}) = \{\varphi \in \mathcal{L}_K \mid \mathcal{M} \models \varphi\}$):

Definition 2.1 *A model $\mathcal{M}$ is a model of minimal knowledge for $I \subseteq \mathcal{L}_K$ in the logic $\mathcal{S}$, if $\mathcal{M}$ is a model for I in $\mathcal{S}$ and for every model $\mathcal{M}'$ of I in $\mathcal{S}$, $Th(\mathcal{M}') \cap \mathcal{L} \not\subset Th(\mathcal{M}) \cap \mathcal{L}$.*

We say that a logic $\mathcal{S}$ is a logic of minimal knowledge if for every theory $I \subseteq \mathcal{L}_K$, every model for I in $\mathcal{S}$ is a model of minimal knowledge for I in $\mathcal{S}$.

In [1] minimal knowledge is applied to modal logic S5 and in [5] a possible-world semantics for this logic is given as follows. A model $\mathcal{M}$ is constituted by a set of interpretations: a sentence $K\varphi$ is true in a world $w \in \mathcal{M}$ if φ is true in all $w' \in \mathcal{M}$. This amounts to considering the so-called universal S5 models, i.e. connected structures whose accessibility relation is reflexive, symmetric and transitive. However, not every such structure that satisfies the initial assumptions I of the agent is taken into consideration: the interesting models are the minimal ones, namely those which do not have any proper superset satisfying I. In other words, a S5-model is minimal if it satisfies I and cannot be extended by adding a new possible world. Therefore, minimization of knowledge is obtained by maximizing the set of possible worlds, sometimes explained as maximizing ignorance. The non-monotonic character of this construction becomes evident when looking at the case in which I is the empty set. In this case one can conclude $\neg Kp$ for every atomic sentence p.

This idea has been further developed in [5, 6] where it is embedded in a bimodal logic that combines minimization of knowledge with justified assumptions.

In [16] the minimization of knowledge is formulated in terms of a preference criterion on Kripke models which differs from the one shown in [5]: in particular, a different class of models is taken into consideration, that is the class of models characterizing modal logic S4F. However, this way of minimizing knowledge does not correspond to the minimization of objective facts, so this logic is not a logic of minimal knowledge in the sense of the definition above.

3 McDermott and Doyle's logics

The first attempt to the definition of a modal non-monotonic formalism is due to McDermott and Doyle [11, 10]. Their approach is based on a fixed point equation applied to the consequence operator of a monotonic modal logic. This equation is the general scheme for defining expansions, namely possible sets of sentences representing the knowledge of an agent reasoning intospectively from an initial body of knowledge. Given a modal logic $\mathcal{S}$, a consistent set of formulas T is an *$\mathcal{S}$-expansion* for a set of initial knowledge

$I \subseteq \mathcal{L}_K$ if

$$T = Cn_{\mathcal{S}}(I \cup \{\neg K\varphi \mid \varphi \notin T\}), \tag{1}$$

where $Cn_{\mathcal{S}}$ is the consequence operator in (classical) modal logic $\mathcal{S}$.

The resulting consequence operator is defined as the intersection of all $\mathcal{S}$-expansions for I. Such operator is in general non-monotonic: thus for every modal logic $\mathcal{S}$, the (non-monotonic) modal logic $\mathcal{S}_{MDD}$ is obtained by means of equation (1).

The McDermott and Doyle's family of non-monotonic modal logics has been extensively studied [12, 8, 7, 9]: McDermott [10] analyzed the case of $\mathcal{S} =$ S5 and found out that the resulting logic $\mathrm{S5}_{MDD}$ is monotonic, in the sense that the intersection of all S5-expansions of a theory I is exactly the set of consequences of I in monotonic S5. Schwarz [13] proved the equivalence of Moore's autoepistemic logic [12] with logic $\mathrm{KD45}_{MDD}$. He also defined in [14] a preference semantics for McDermott and Doyle's family, thus giving a true possible-world semantics characterization of this class of modal non-monotonic formalisms.

This result allows us to study properties of logics $\mathcal{S}_{MDD}$ by reasoning on Kripke structures, which is often easier than analyzing sets of modal formulae. From this semantic viewpoint it is easy to show that $\mathrm{S5}_{MDD}$ is not the only degenerate (i.e. monotonic) case for logics $\mathcal{S}_{MDD}$. More specifically, it can be shown that for every normal modal logic $\mathcal{S}$, characterized by a class of Kripke models whose accessibility relation is symmetric, its non-monotonic variant $\mathcal{S}_{MDD}$ is equivalent to monotonic logic S5. Therefore there is a whole class of logics in the McDermott and Doyle's family which degenerate to the case of monotonic S5, precisely all logics containing modal axiom schema B, i.e. $\neg K \neg K\varphi \supset \varphi$.

For our purposes, the most important result concerning McDermott and Doyle's logics is the following:

Proposition 3.1 *None of $\mathcal{S}_{MDD}$ logics is a logic of minimal knowledge.*

This can be easily seen by looking at the theory $I = \{\neg K \neg K p\}$: given any modal logic $\mathcal{S}$, I has at least two $\mathcal{S}$-expansions, T_1 and T_2, such that $T_1 \cap \mathcal{L} = Cn(\{p\})$ and $T_2 \cap \mathcal{L} = Cn(\emptyset)$, where Cn is the consequence operator in propositional logic. Since $Cn(\emptyset) \subset Cn(\{p\})$, $\mathcal{S}_{MDD}$ is not a logic of minimal knowledge.

4 Modal logics of minimal knowledge

In this section we consider the family of ground non-monotonic logics. This family is defined by a slight modification of (1), by bounding introspection in the right-hand side of the equation to modal-free formulas only.

Given a normal modal logic $\mathcal{S}$, a consistent set of formulas T is a *ground $\mathcal{S}$-expansion* for a set $I \subseteq \mathcal{L}_K$ if

$$T = Cn_{\mathcal{S}}(I \cup \{\neg K\varphi \mid \varphi \in \mathcal{L} \backslash T\}). \tag{2}$$

This equation defines a family of logics $\mathcal{S}_G$ called *ground* non-monotonic logics. In [9] a property of minimality has been shown for these logics which in turn implies the property of minimal knowledge.

We now present the semantic characterization of ground non-monotonic logics.

A Kripke model $\mathcal{M}$ is defined as usual by a triple $\langle W, R, V\rangle$, where W is a set (whose elements are called worlds), R is a binary relation on $\mathcal{M}$ (called the accessibility relation on $\mathcal{M}$), and V is a function assigning a propositional valuation to each world $w \in \mathcal{M}$. First we need the following definitions:

Definition 4.1 *Given Kripke models $\mathcal{M} = \langle W, R, V\rangle$, $\mathcal{M}_1 = \langle W_1, R_1, V_1\rangle$, $\mathcal{M}_2 = \langle W_2, R_2, V_2\rangle$, such that $W_1 \cap W_2 = \emptyset$, $\mathcal{M} = \mathcal{M}_1 \odot \mathcal{M}_2$ if $W = W_1 \cup W_2$, $V = V_1 \cup V_2$ and $R = R_1 \cup (W_1 \times W_2) \cup R_2$.*

Definition 4.2 *Given the two Kripke models $\mathcal{M}_1 = \langle W_1, R_1, V_1\rangle$ and $\mathcal{M}_2 = \langle W_2, R_2, V_2\rangle$, $\mathcal{M}_2 \supset_G \mathcal{M}_1$ if $W_1 = W_2$, $V_1 = V_2$ and $R_2 \supset R_1$.*

We can now define the ordering relation on Kripke models.

Definition 4.3 *Given two Kripke models $\mathcal{M}_1$, $\mathcal{M}_2$, $\mathcal{M}_2 \sqsubset_G \mathcal{M}_1$ if there exists a Kripke model $\mathcal{M}$ such that $\mathcal{M}_2 \supset_G \mathcal{M} \odot \mathcal{M}_1$.*

Finally, minimal models are characterized using the $\sqsubset_G$ ordering.

Definition 4.4 *Given a normal modal logic $\mathcal{S}$ characterized by the class of Kripke models $\mathcal{C}$, a model $\mathcal{M} \in \mathcal{C}$ is a* ground $\mathcal{C}$-minimal model *for I if $\mathcal{M} \models I$ and for every model $\mathcal{M}' \in \mathcal{C}$ such that $\mathcal{M}' \models I$, $\mathcal{M}' \not\sqsubset_G \mathcal{M}$.*

We now recall some preliminary definitions (see [9] for further details), to state the theorem that establishes the correspondence between the fixed-point characterization (2) and the semantics based on ground minimal models.

Definition 4.5 *A theory* $T \subseteq \mathcal{L}_K$ *is* stable *if*

1. T *is closed under propositional consequence;*

2. *for every* $\varphi \in \mathcal{L}_K$*, if* $\varphi \in T$ *then* $K\varphi \in T$*;*

3. *for every* $\varphi \in \mathcal{L}_K$*, if* $\varphi \notin T$ *then* $\neg K\varphi \in T$*.*

Definition 4.6 *The* canonical model for a stable theory T *is a Kripke model* $\mathcal{M}_T = \langle W, R, V\rangle$ *such that* W *consists of all propositional valuations in which all formulas from* $T \cap \mathcal{L}$ *are true,* R *is the universal relation on* W *(hence* $\mathcal{M}_T$ *is a S5-model) and* $V(w) = w$ *for every* $w \in \mathcal{M}$*.*

Theorem 4.7 *Given a normal modal logic* $\mathcal{S}$*, characterized by the class of Kripke models* $\mathcal{C}$*, and a theory* $I \subseteq \mathcal{L}_K$*, a stable theory* T *is a* ground $\mathcal{S}$-expansion *for* I *if and only if the canonical model* $\mathcal{M}_T$ *is a ground* $\mathcal{C}$*-minimal model for* I*.*

This theorem provides a semantic characterization for the family of ground non-monotonic logics $\mathcal{S}_G$. It is a special case of Shoham's preference semantics [17]. Moreover, the preference criterion obtained through the definition of the relation $\sqsubset_G$ can be seen as a stronger version of the preference criterion found by Schwarz for Mc Dermott and Doyle's logics [14].

This semantic characterization has allowed us to find interesting properties of ground non-monotonic logics. In particular:

- none of these logics collapses into a monotonic logic (in particular for $\mathcal{S}$=S5 the logic defined in [1] is obtained, as stated in [15]);

- all logics $\mathcal{S}_G$ are insensitive to the axiom schema D, in the sense that for every normal modal logic $\mathcal{S}$, every istance of schema D is a tautology in $\mathcal{S}_G$ (this property holds for logics $\mathcal{S}_{MDD}$ as well);

- given any two normal modal logics $\mathcal{S}$, $\mathcal{S}'$, each containing the axiom schema D, $\mathcal{S}$ is equivalent to $\mathcal{S}'$ if and only if their ground non-monotonic extensions $\mathcal{S}_G$, $\mathcal{S}'_G$ are equivalent. Namely, with the exception of schema D, every normal modal logic $\mathcal{S}$ produces a different non-monotonic logic $\mathcal{S}_G$. Therefore the family $\mathcal{S}_G$ shows more variety than the family $\mathcal{S}_{MDD}$ (see [9], par. 11.2).

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